Mathematical Psychology meets Psychometrics (again)

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1. What have we got?

- Item Response Theory (and Model)

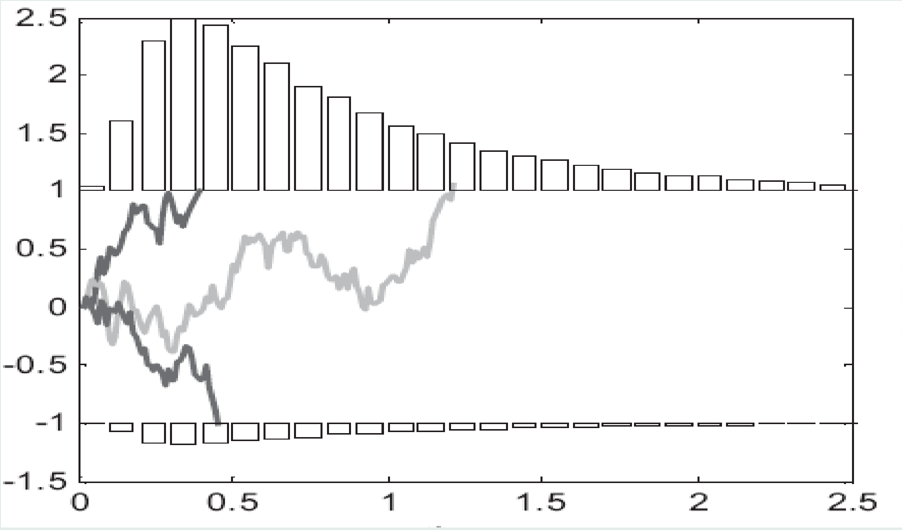
$$\Pr(X_{pi} = 1) = \pi_{pi}$$

- Regression:

$$\ln\left(\frac{\pi_{pi}}{1 - \pi_{pi}}\right) = \theta_p - \delta_i$$

2. What have they got: Diffusion Model

An Item Response Theory Model



which looks horrible

$$f(x,t) = \frac{\pi\sigma^2}{(a_+ + a_-)^2}\left[\exp\left([[a_+ + a_-]x - a_-]\frac{\mu}{\sigma^2} - t\frac{\mu^2}{2\sigma^2}\right)\right.$$

$$\sum_{m=1}^{\infty} m \sin\left(\frac{\pi m[[a_+ + a_-]x - 2a_- x + a_-]}{a_+ + a_-}\right)$$

$$\left.\exp\left(-\frac{\pi^2\sigma^2 m^2}{2(a_+ + a_-)^2}t\right)\right]$$

2.1. Unbiased Diffusion

It is easily seen that if both boundaries are equal ($a_+ = a_- = a$) things get simpler:

$$f(x,t) = \frac{\pi\sigma^2}{(2a)^2}\left[\exp\left([2ax - a]\frac{\mu}{\sigma^2} - t\frac{\mu^2}{2\sigma^2}\right)\right.$$
$$\left.\sum_{m=1}^{\infty} m \sin\left(\frac{\pi m}{2}\right)\exp\left(-\frac{\pi^2\sigma^2 m^2}{2(2a)^2}t\right)\right]$$

2.2. Volatility drives the boundary

$$f(x,t) = \pi \left[\exp\left([2x-1]\frac{\mu}{2\sigma} - t\frac{\mu^2}{2\sigma^2} \right) \sum_{m=1}^{\infty} m \sin\left(\frac{\pi m}{2}\right) \exp\left(-\frac{\pi^2 m^2}{2} t \right) \right]$$

Some properties

- $X \perp\!\!\!\perp T$
- $P(X = 1) = \frac{\exp\left(\frac{\mu}{\sigma}\right)}{1+\exp\left(\frac{\mu}{\sigma}\right)}$
- $\mathcal{E}(T) = \frac{\sigma}{2\mu}\frac{\exp\left(\frac{\mu}{\sigma}\right)-1}{\exp\left(\frac{\mu}{\sigma}\right)+1}$
- $\frac{\mu}{\sigma} = \frac{2\mathcal{E}(X)-1}{2\mathcal{E}(T)}$

3. IRT parameterization

$$\mu_{pi} = \theta_p - \beta_i$$

and

$$\sigma_{pi} = \alpha_i$$

or

$$\mu_{pij} \sim \theta_p - \beta_i + \epsilon_{pij} \quad , \epsilon_{pij} \overset{iid}{\sim} \mathcal{N}(0,1)$$

4. Estimation

$$f(\mathbf{x}, \mathbf{t}) = \prod_p \prod_i \pi \left[\exp\left([2x_{pi} - 1] \frac{\theta_p - \delta_i}{2\sigma_i} - t_{pi} \frac{(\theta_p - \delta_i)^2}{2\sigma_i^2} \right) \right.$$
$$\left. \sum_{m=1}^{\infty} m \sin\left(\frac{\pi m}{2}\right) \exp\left(-\frac{\pi^2 m^2}{2} t_{pi} \right) \right]$$

- JML
- MML: Likelihood is analytical
- adaptive estimation:

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5. Discussion